**Single Parameter Scaling Theory**

Consider the ferromagnetic spin problem. It depends on just two parameters: T, h. If we know how they change upon rescaling, then we can, from the Kadanov thermodynamic relations, upon the critical exponents.

**Basic Idea Applied to Transport**

Such an approach could be useful to our transport problem here. The rescaling would seem to be more literal than above. So we might consider length dependent transport (or not) quantities and consider how they scale as we extend the length of the system. Now in general transport would depend on various quantities like: **χ** = EF, τF, ℓ, ni, band structure/anisotropy, B, etc., besides length. And in general, a length dependent transport related quantity, q, would follow some such equation as:



But we may hope to identify a set of quantities qn(L), like g(L), or maybe getting ahead of ourselves <g(L)>n, possibly quantities like En(L), or IPR(L) = ΣnIPR(ψn(L)) whatever. Then the question is can we write a self-consistent relationship for these, like is done for T and h? One possibility is to write:



where fn are pure number function so that all physically meaniful parameters appear only within the qj(L0). This is a necessary condition for our ultimate aim, but not sufficient (see that other file for explorations of the difference). What we want is a scaling relationship like we have in SM for the Ferromagnet problem:



From here a general RG equation would emerge:



and,



The function βn(x) would be presumed to be pure #. Hopefully, also, g(L) would be one of these parameters, or could be obtained from them…in which case I guess it could be considered one of them. Then we might look for a fixed point, which we could call qj\*. This would be a scale invariant point, and would indicate that these quantities are invariant at the fixed point. Is there any reason, at this point, why we might suppose g, or any of its moments, or any other quantity like IPR(L) to have such a point? Regardless, we might linearize around this fixed point, and find:



then define new variables:



Multiply through,



and our q’s would be:



Solving for initial conditions:



But whatever. So then we have:



So the ξα are various relevant length scales, and λα the scaling exponents. Some of these eigenvalues λα will be positive, some negative. And there doesn’t seem to be a reason to expect any given outcome. The positive ones would be ‘relevant’, and the negative ones ‘irrelevant’. If these are all negative except for one, then we may say that, at least close to the critical point, we have a one parameter scaling theory. And there would be only one relevant length scale near the critical point. It is a question how close to the critical point we’d need to be, or how large our system would need to be, as the other parameters would be expected to play a role if we’re not close enough, large enough. Experimentally we could test this hypothesis by changing all of these parameters χ and observing whether q, for instance, lies on a single curve. But, well, remember that this may be a multi-branch function, like how g = f(L/ξ), which has different forms on each side of the criticial point.

**Do <g(L)>n constitute an at least multi-parameter scaling theory, MPST?**

So first question is, ‘does a multi-parameter scaling theory hold?’ I seem to have mixed messages on this, mainly pertaining to the fact that g is a statistical variable with nontrivial cumulants, <g>n. So one might provisionally hypothesize that <g(L)>n all form self-consistent set. But the results of the NLσM, which showed length dependent cumulants at the critical point seemed to rule that out. Suslov and other guy argued for other possibilities – one that the NLσM was simply getting worse with larger ε, since it did justify single parameter scaling when ε = 0. Another possibility was a compromise position that accepted the results of NLσM but interpreted the divergence as due to a Cauchy-like critical distribution in 2 + ε dimensions. But even this would seem to concede, to me, that the critical distribution wasn’t invariant, but merely scaled to a length-independent result, which still seems at odds with a real fixed point scaling theory. Hmmm….

**Do <g(L)>n constitute a SPST?**

Even if we concede they are a scaling theory, are they a single parameter scaling group? We’d have to test that all cumulants depend on only one thing, which we might as well take to be <g> itself. Analytical results in the weak disorder regime seem to bear it out so far, at least in Q1D. I’ve seen people say that they’ve been able to verify that higher moments of P(g) can be written as function of <g> itself.

Markos says that people have examined quartiles of the probability distribution P(g), and shown that they obey single parameter scaling.

I have read somewhere that it might be that single parameter scaling only works for states with energies near the band center, for not-too-strong-disorder, and not too far from the transition (I guess this makes sense sort of since we our fixed point linearization is only accurate near the fixed point).

**Does <g(L)>, or <lng(L)>, or gmost probable, etc., obey SPST?**

MacKinnon claims that he finds <g> doesn’t really obey single parameter scaling. And then he says that NLsM model would then be illegitimate because it’s a model of <g>. This is how it makes such bad critical exponent predictions, he says. I’m not sure where in the NLsM we make assumption that it is <g> we’re working with, rather than say gp, or something else. Final note, it would seem we could still frame, in fact more naturally even, the SPS in terms of <lng>, rather than <g>, however. See Lerner 1991 for a discussion of this point. But then Mac also seems to say that there is evidence of a SPS for non-zero B, at least near band center, and not too far, etc. He says that he gets the same critical exponents for zero and non-zero B. ES say that highly anisotropic systems obey SPS, though perhaps with different critical points, but still same critical exponents as isotropic ones.

**What are limits of scaling theory?**

Some people I don’t recall say that they find SPS breaks down in the large disorder regime. I suppose this isn’t shocking per seʹ, but it would seem to have implications for a large disorder scaling equation in even 1D, and especially Q1D, 3D.

**Miscellaneous result**

Angus/MacKinnon derive a lower bound for the scaling exponent (assuming single parameter scaling theory, and making some other seemingly reasonable assumptions). They find **ν > 2/d**.